



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 33: SGM and Wave Propagation

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ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
3. Pile Foundations
4. Well Foundations
5. SSI for Deep Foundations
6. Miscellaneous

Module - 5

SSI for Deep Foundations



Module - 5: Chapters

1. Introduction to Soil-Structure Interaction
2. Effects of Soil-Structure Interaction
3. SGM and Wave Propagation
4. Dispersion & Attenuation of Waves, Damping
5. Ground Response Analysis
6. Soil-Pile Interaction

Chapter-3: SGM and Wave Propagation

1. Strong Ground Motion (SGM)
2. Wave Propagation

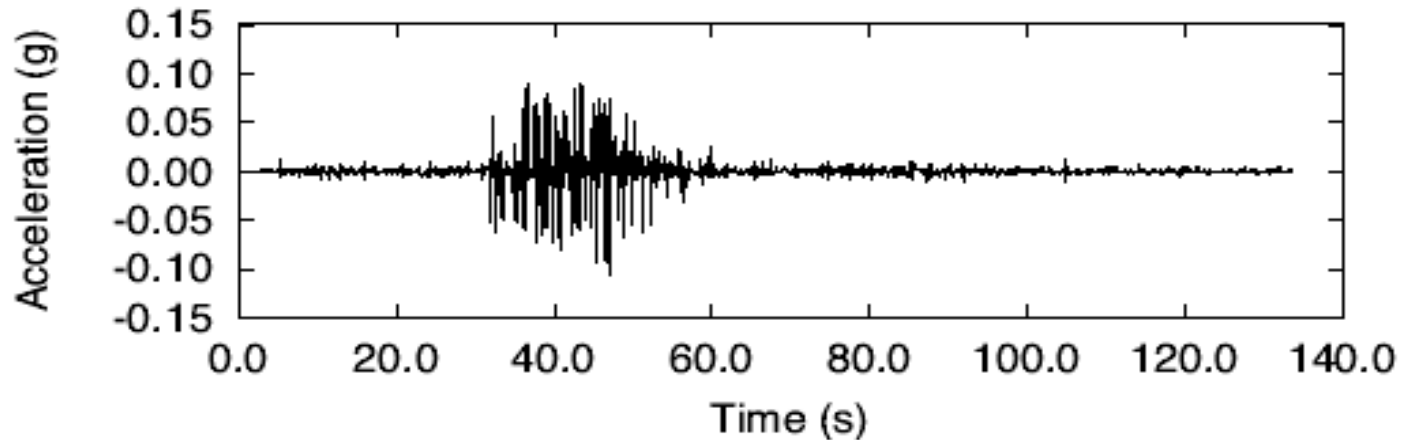
Strong Ground Motion (SGM)

- **Vibrations of Earth**
 - Period: Milliseconds to Days
 - Amplitude: Nanometers to Meters
- **Micro-Seismic Activity** (weak vibrations)
 - Greater Importance to Seismologists
- **Strong Ground Motion**
 - Very important for Earthquake Engineers
 - Quantitative ways of describing SGM?

Strong Ground Motion (SGM)

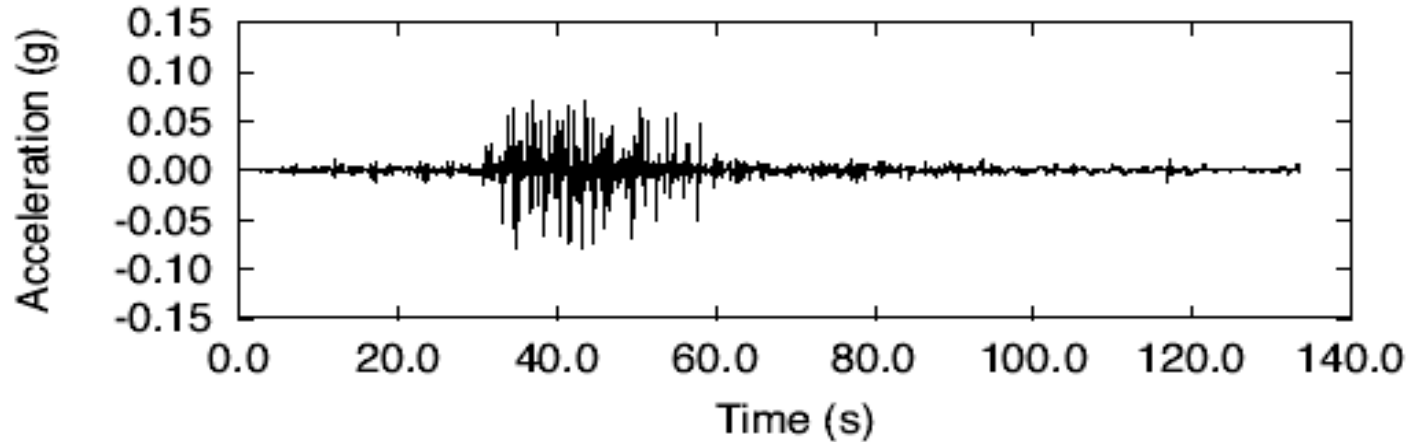
- The ground motions due to an EQ
 - May be quite complicated
 - Represented by 3 components of translation and 3 components of rotation which are usually neglected
 - Most commonly 3 translational component (Such as N-S, E-W & Vertical) are measured
 - Typical record contains a lot of information
 - 3 Components of Bhuj EQ (next slides)

Components of Bhuj EQ – Recorded at Passport Office Ahmedabad



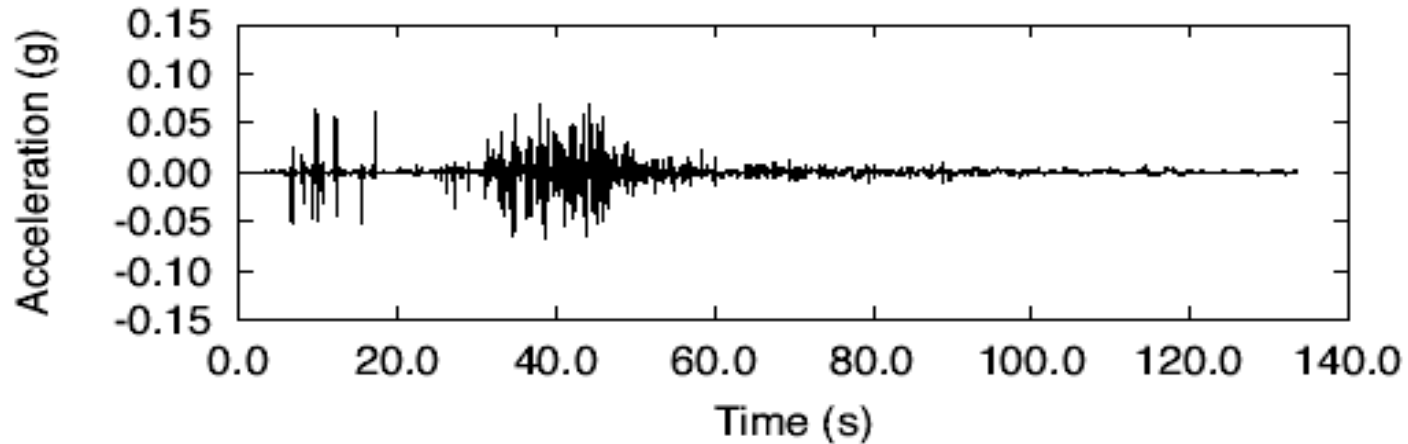
Longitudinal Component (N78° E)

Components of Bhuj EQ – Recorded at Passport Office Ahmedabad



Transverse Component (N12° W)

Components of Bhuj EQ – Recorded at Passport Office Ahmedabad



Vertical Component

Strong Ground Motion (SGM)

it is not necessary to reproduce each time history exactly to describe the ground motion adequately for engineering purposes. However, the characteristics of the ground motion that are of engineering significance needed to be described and a number of ground motion parameters that reflect those characteristics needed to be identified.

SGM – Characteristics of EQ Motion

- **Amplitude**
 - Usually described by PGA
- **Frequency Content**
 - Described by the Fourier Spectra
- **Duration of the motion**
 - Usually in Seconds

Chapter-3: SGM and Wave Propagation

1. Strong Ground Motion (SGM)
2. Wave Propagation

Wave Propagation

- Generation of waves in a continuous medium Due to disturbance in the Medium are analogous to the waves created when a pebble is dropped in a pond.
- An EQ produces a motion of the ground by the passage of stress waves that originate from the rupture of the stressed earth mass.
- Waves may also be generated **both at the surface and within the earth** by man-made vibrations.
- **Wave velocities depend upon the elastic properties** of the medium through which they travel

Wave Propagation Theories

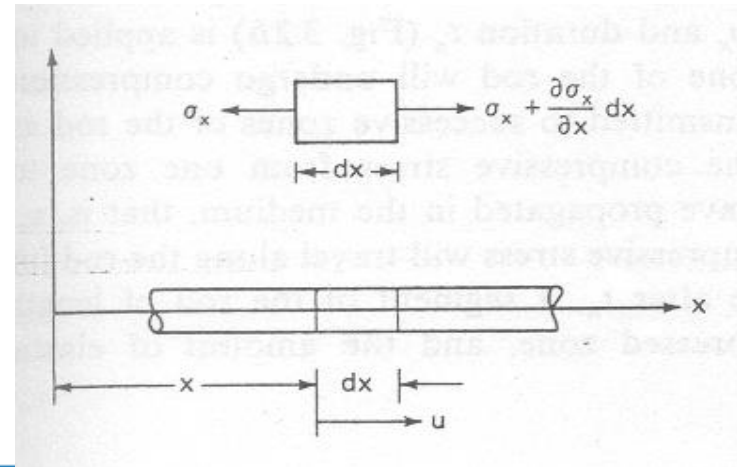
- 1. One-Dimensional Wave Propagation in an Elastic Rod**
- 2. Three-Dimensional Wave Propagation in an Elastic Infinite Medium**
- 3. Wave Propagation in a Semi-Infinite Elastic Half-Space**

1-D Wave Propagation in an Elastic Rod

Considering the longitudinal vibration of an elastic rod with cross sectional area A , Young's Modulus E and mass density ρ , force equilibrium under free vibration lead to:

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} = \frac{1}{\rho} \frac{\partial \sigma_x}{\partial x} = \frac{E}{\rho} \frac{\partial^2 \mathbf{u}}{\partial x^2} = v_r^2 \frac{\partial^2 \mathbf{u}}{\partial x^2} \quad (1) \quad \text{where } v_r^2 = \frac{E}{\rho}$$

where v_r is the Longitudinal Wave propagation velocity in a rod. Eq. (1) is defined as 1-D wave equation.



3-D Wave Prop. in an Elastic Infinite Medium

It is assumed that the infinite medium through which waves propagate is homogenous and isotropic. The 3-D equations of motion of an elastic solid are expressed in terms of displacements. For this stress-strain and strain-displacements relationship are used. Manipulation of Equations and subsequent solution lead to:

$$\frac{\partial^2 \epsilon_v}{\partial t^2} = v_c^2 \nabla^2 \epsilon_v$$

Where v_c is the velocity of compression waves

$$v_c^2 = \frac{\lambda + 2\mu}{\rho} = \frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)} = \frac{K}{\rho}$$

3-D Wave Prop. in an Elastic Infinite Medium

For $\nu = 0$, $v_c^2 = E/\rho$ i.e. v_c is equal to the velocity of compression wave propagation in rod v_r .

For $\nu > 0$, $v_c > v_r$.

The other solutions of the equations lead to shear wave velocity of propagation given by

$$v_s^2 = \frac{\mu}{\rho} = \frac{E}{\rho} \cdot \frac{1}{2(1+\nu)}$$

Ratio of v_c/v_s is given by

$$\frac{v_c}{v_s} = \sqrt{\frac{2(1-\nu)}{(1-2\nu)}}$$

3-D Wave Prop. in an Elastic Infinite Medium

An infinite elastic medium can sustain two kinds of waves:

1. Compression wave (with velocity v_c), also called Primary Wave, P Wave, Irrotational Wave.
2. Shear wave (with velocity v_s), also called Secondary Wave, S Wave, Distortional Wave, Voluminal Wave.

The two waves, which represent different types of body motions, travel at different velocities. It may be noted that the particle motion associated with the compression wave in the rod and that in the infinite medium is the same, but the wave propagation velocities are different.

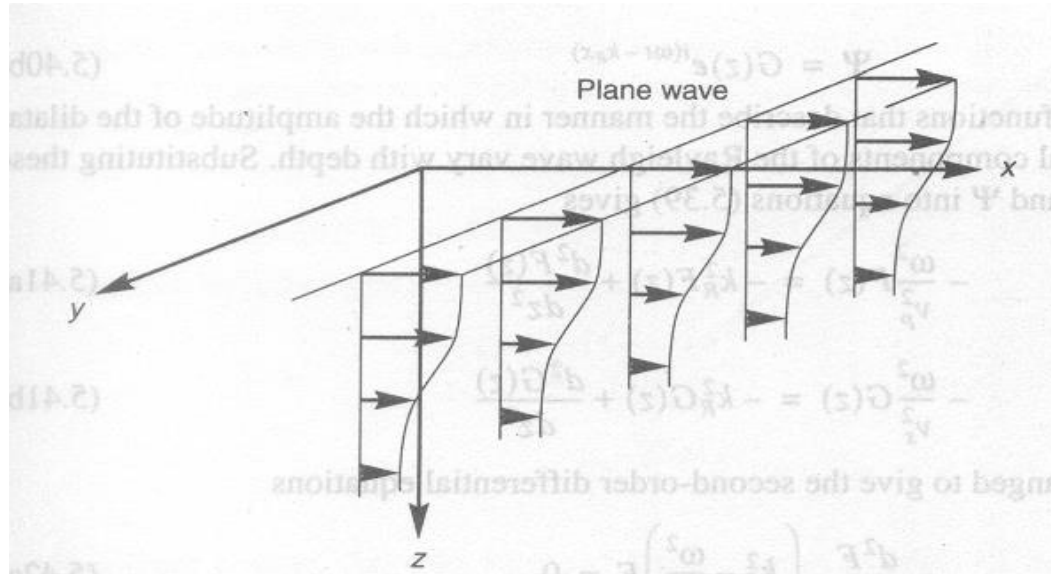
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3-D Wave Prop. in an Elastic Infinite Medium

Since $v_c > v_r$, the compression wave travels faster in the infinite medium than in a rod. The second type of wave motion (S wave) propagates at the same velocity in the both medium. Following table shows the velocity of wave propagation in compression and shear for different materials.

Soil	ρ (kg/m ³)	v_c (m/s)	v_s (m/s)
Moist Clay	1.8×10^3	1500	150
Loess at natural moisture	1.67×10^3	800	260
Dense sand and gravel	1.70×10^3	480	250
Fine-grained sand	1.65×10^3	300	110
Medium-grained sand	1.65×10^3	550	160
Medium-sized gravel	1.8×10^3	750	180

Wave Propagation in a Semi-Infinite Elastic Half-Space

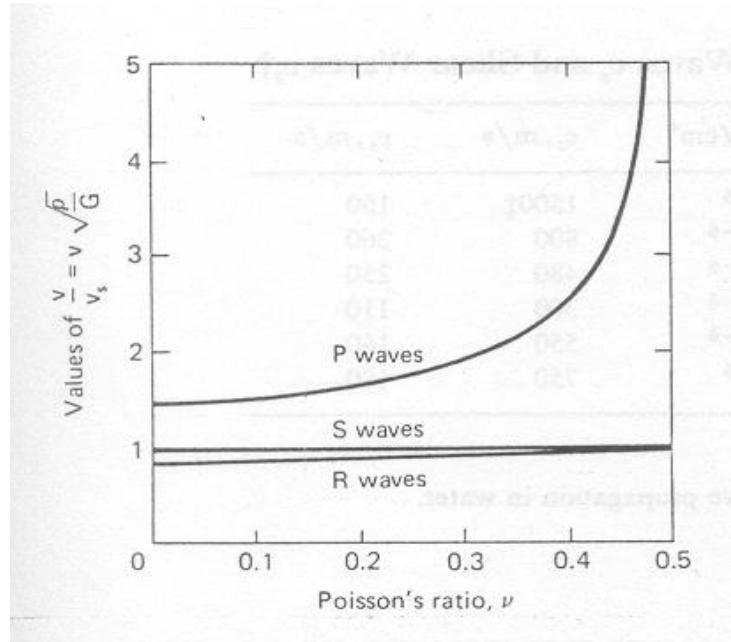


Motion induced by a typical plane wave that propagates in the x -direction. Wave motion does not vary in the y direction.

Wave Propagation in a Semi-Infinite Elastic Half-Space

In practice, the solutions for wave propagation in an infinite elastic body have little significance. Since foundations are supported on the soil, the boundary conditions approximating this situation are those of **a semi-infinite half space**. It is assumed that the medium is homogeneous, isotropic, and elastic. In an elastic infinite medium, it was found that there are two types of body waves, the compression wave and the shear wave. In an elastic half space, it will be seen that another wave, **the Rayleigh wave**, shows up. The motion of a Rayleigh wave is confined to a zone near the boundary of the half space.

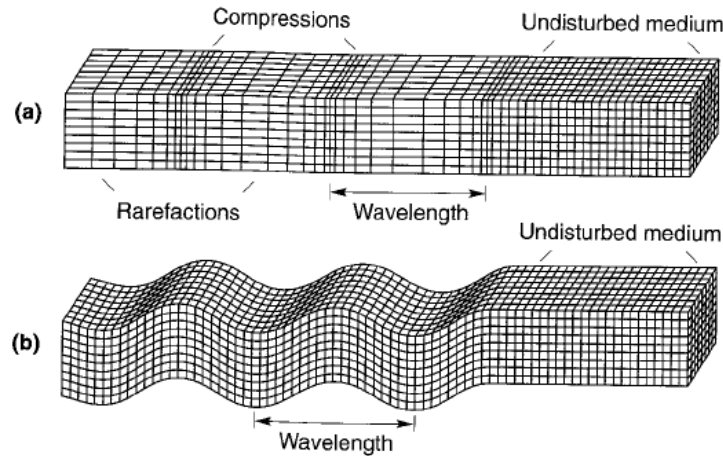
Variation in Wave Velocities with Poisson's ratio (ν)



$$\frac{v_c}{v_s} = \sqrt{\frac{2(1-\nu)}{(1-2\nu)}}$$

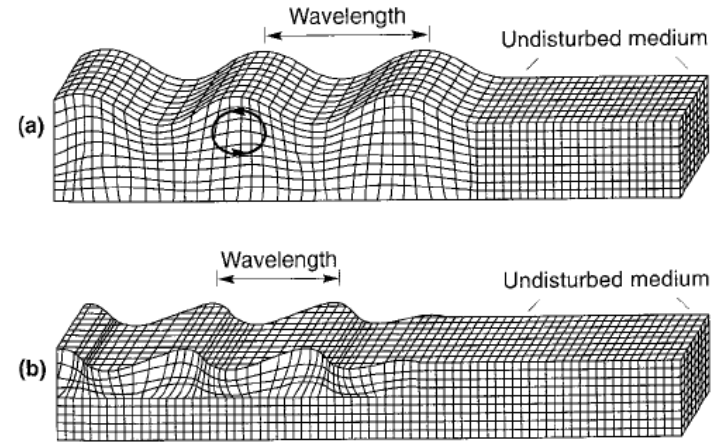
Relation between Poisson's ratio ν and velocities of propagation of compression (P), Shear (S) and Rayleigh (R) waves in a semi-infinite elastic Half-Space

Directions of Wave Propagation & Particle Motion



Body Waves

- a. Compression (P) waves
- b. Shear (S) waves



Surface Waves

- a. Rayleigh (R): by P and SV waves
- b. Love (L): by SH waves

References:

1. Prakash S. (1981), Soil Dynamics, McGraw-Hill Book Company, New York
2. Kramer S.L. (1996). Geotechnical Earthquake Engineering, Prentice Hall, Upper Saddle River, NJ, USA

Thank You

